① $\int \frac{5}{x}\,dx = 5\int \frac{1}{x}\,dx = 5\int \frac{u'}{u} = 5\ln|u| + C$

$u = x$

$du = 1$

$\boxed{= 5\ln|x| + C}$

② $\int \frac{1}{x+2}\,dx$ $\quad u = x+2$, $du = 1$ $\quad = \int \frac{u'}{u} = \ln|u| = \ln|x+2| + C$

③ $\int \frac{x^2-4}{x}\,dx = \int \frac{x^2}{x} - \frac{4}{x} = \int x - 4\int \frac{1}{x}$

$= \frac{1}{2}x^2 - 4\ln|x| + C$

④ $\int \frac{x^2}{3-x^3}\,dx$ $\quad u = 3-x^3$, $du = -3x^2$

$-\frac{1}{3}\int \frac{u'}{u} = -\frac{1}{3}\ln|3-x^3| + C$

⑤ $\int e^{-x^4}(-4x^3)\,dx$ $\quad u = -x^4$

$du = -4x^3$

$\int e^u\,du = e^u + C = e^{-x^4} + C$

⑥ $\int x^2 e^{\frac{x^3}{2}}\,dx$ $\quad u = \frac{1}{2}x^3$

$du = \frac{3}{2}x^2$

$\frac{2}{3}\int e^u\,du = \frac{2}{3}e^u + C = \frac{2}{3}e^{\frac{x^3}{2}} + C$

⑦ $\int \frac{e^{\frac{1}{x^2}}}{x^3}\,dx$ $\quad u = \frac{1}{x^2} = x^{-2}$

$du = -2x^{-3} = \frac{-2}{x^3} = -2 \cdot \frac{1}{x^3}$

$-\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{\frac{1}{x^2}} + C$

⑧ $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}$ $\quad u = e^x - e^{-x}$

$du = e^x + e^{-x}$

$\int \frac{u'}{u} = \ln|u| + C = \ln|e^x - e^{-x}| + C$

⑨ $\int 3^x \, dx$

$\int a^u \, du = \frac{1}{\ln a} \cdot a^u + C$

$u = x$
$du = 1$

$\int 3^u \, du = \frac{1}{\ln 3} \cdot 3^x + C$

⑩ $\int x\left(5^{-x^2}\right) dx$ $\quad u = -x^2$
$du = -2x$

$-\frac{1}{2} \int 5^u \, du = -\frac{1}{2} \cdot \frac{1}{\ln 5} \cdot 5^{-x^2} + C$

$= -\frac{1}{2\ln 5} \cdot 5^{-x^2} + C$

(11) $\int (3-x)\, 7^{(3-x)^2}\, dx$

$u=(3-x)^2$

$du = 2(3-x)\cdot(-1) = -2(3-x)$

$-\frac{1}{2}\int 7^u\, du = -\frac{1}{2}\cdot\frac{1}{\ln 7}\cdot 7^{(3-x)^2} + C$

$= -\frac{1}{2\ln 7}\cdot 7^{(3-x)^2} + C$

(12) $\int \frac{3^{2x}}{1+3^{2x}}$ $u = 1+3^{2x}$

$du = \ln 3\,(3^{2x})\cdot 2$

$\frac{1}{2\ln 3}\int \frac{u'}{u} = \frac{1}{2\ln 3}\cdot \ln|1+3^{2x}| + C$

(13) $\int \frac{7}{16+x^2}\,dx = 7\int \frac{1}{16+x^2}\,dx$ $\quad a=4$, $u=x$

$$= \frac{7}{4}\arctan\frac{x}{4}+C$$

x not 1 so use "u" substitution

(14) $\int_{-\frac{1}{2}}^{0} \frac{x}{\sqrt{1-x^2}}\,dx$

$u=1-x^2$

$du=-2x$

$$-\frac{1}{2}\int_{-\frac{1}{2}}^{0}\frac{u'}{\sqrt{u}} = -\frac{1}{2}\int_{-\frac{1}{2}}^{0} u^{-\frac{1}{2}}\cdot du = -\frac{1}{2}\cdot 2u^{1/2}$$

$$= -1\cdot\sqrt{1-x^2}\Big]_{-\frac{1}{2}}^{0}$$

$$= -1 - .866$$

$$= -.134$$

(15) $\displaystyle\int \frac{dx}{x\sqrt{4x^2-9}}$ $\quad u = 2x$, $a = 3$

to cancel the 2 in bottom $\rightarrow$ $\displaystyle 2\cdot\int \frac{1}{(2x)\sqrt{(2x)^2-3^2}}\,dx$

$$\frac{2}{3}\operatorname{arcsec}\frac{|2x|}{3} + C$$

(16) $\displaystyle\int \frac{x^3}{x^2+1}$

LONG DIVISION: $x^2+0x+1 \overline{\big)\, \cancel{x^3}+0x^2+0x+0}$, quotient $x + \dfrac{-1x}{x^2+1}$

$-\ \cancel{x^3}+0x^2+1x$

$-1x+0$ remainder

$\displaystyle\int x - \int \frac{x}{x^2+1}$ $\quad$ u substitution, $u = x^2+1$, $du = 2x$

$$\frac{1}{2}x^2 - \frac{1}{2}\ln|x^2+1| + C$$