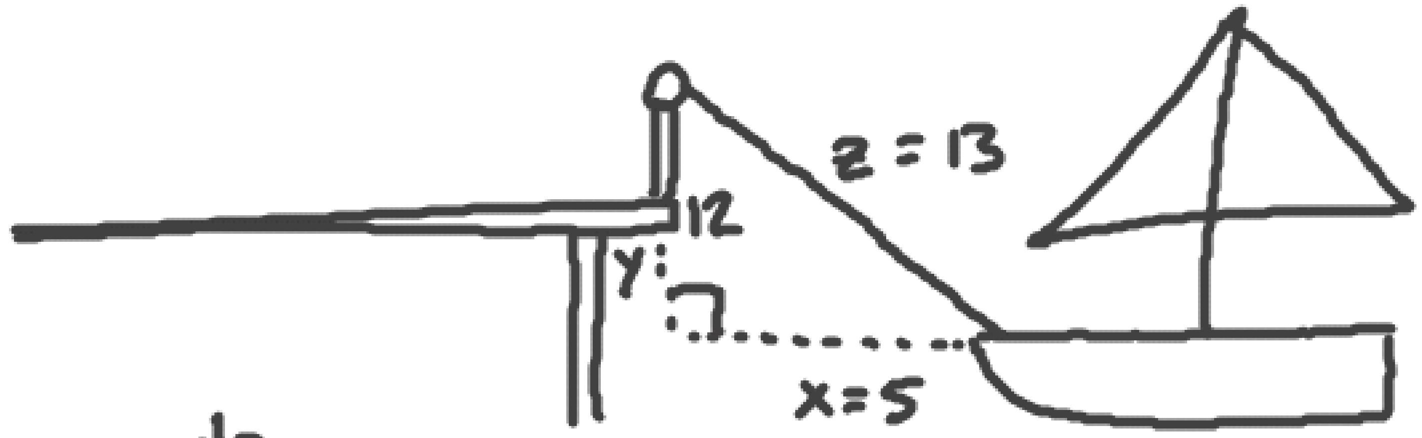
1.



$\frac{dz}{dt} = -4$ ft/sec

$z = 13$

$y = 12$

$\frac{dy}{dt} = 0$

$x = 5$

$\frac{dx}{dt} = ?$

$x^2 + 12^2 = 13^2$

$x^2 = 169 - 144$

$x^2 = 25$

$x = 5$

$\frac{d}{dt}\left[x^2 + 12^2 = z^2\right]$

y won't change so it stays 12

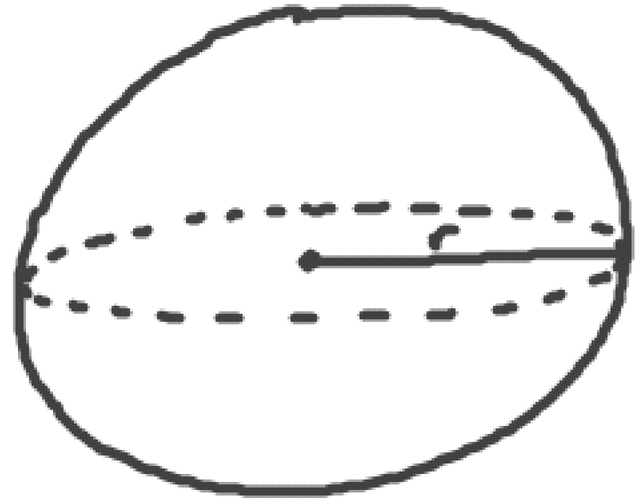
$2x\frac{dx}{dt} + 0 = 2z\frac{dz}{dt}$

$2(5)\frac{dx}{dt} = 2(13)(-4)$

$\frac{10\frac{dx}{dt}}{10} = \frac{-104}{10}$

$\frac{dx}{dt} = -10.4$ ft/sec

2.



$\frac{dr}{dt} = 2$ in/min.

$r = 6$

$\frac{dV}{dt} = ?$

$V = \frac{4}{3}\pi r^3$

$\frac{d}{dt}\left[V = \frac{4}{3}\pi r^3\right] = \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(6)^2(2)$

$\frac{dV}{dt} = 288\pi$ inches^3/min.

3. $f(x) = 12x^3 + 3x$

$f'(x) = 36x^2 + 3$

$f''(x) = 72x$

4. $f(x) = 2 - \frac{3}{x} = 2 - 3x^{-1}$

$f'(x) = 3x^{-2}$

$f''(x) = -6x^{-3} = -\frac{6}{x^3}$

5. $f(x) = 2\sqrt{x} = 2x^{1/2}$

$f'(x) = x^{-1/2}$

$f''(x) = -\frac{1}{2}x^{-3/2} = -\frac{1}{2\sqrt{x^3}}$

6. $\ln \frac{x^3y^2}{z^4}$

7. $\ln\left(\frac{x}{(x+1)(x-1)}\right)^2$

8. $\ln\sqrt{\left(\frac{x^2+1}{(x+1)(x-1)}\right)^3}$

9. $\ln x + \ln y - \ln z$

10. $3\left[\ln(x^2+1) - 3\ln x\right]$

11. $\frac{1}{3}\ln(a^2+1)$

12. $\ln(2x^2+1)$

$\frac{d}{du}[\ln u] = \frac{u'}{u}$

$u = 2x^2+1$

$u' = 4x$

$$\frac{d}{dx}[\ln(2x^2+1)] = \frac{4x}{2x^2+1}$$

13. $\ln\sqrt{x^2-4} = \frac{1}{2}\ln(x^2-4)$

$u = x^2-4$

$u' = 2x$

$$\frac{1}{\cancel{2}}\left[\frac{\cancel{2}x}{x^2-4}\right] = \frac{x}{x^2-4}$$

14. $\ln\left(\frac{2x}{x+3}\right)$

$u = \frac{2x}{x+3}$ $\quad u' =$ quotient rule

$$\frac{2(x+3) - 1(2x)}{(x+3)^2} = \frac{\cancel{2x} + 6 - \cancel{2x}}{(x+3)^2}$$

$$u' = \frac{6}{(x+3)^2}$$

$$\frac{6}{(x+3)^2} \div \frac{2x}{x+3}$$

$$\frac{{}^{3}\cancel{6}}{\cancel{(x+3)^2}\,(x+3)} \cdot \frac{\cancel{x+3}}{\cancel{2}x} = \frac{3}{x(x+3)}$$

15. $\dfrac{\ln t}{t}$ quotient rule

$\frac{d}{dt}[\ln t] = \dfrac{1}{t}$

$$\frac{\underbrace{\frac{1}{t}\cdot t}_{=1} - 1\cdot \ln t}{t^2} = \frac{1-\ln t}{t^2}$$

16. $f(x) = e^{2x}$ $\frac{d}{dx}[e^u] = e^u \cdot u'$

$e^{2x} \cdot 2 = 2e^{2x}$

17. $y = e^{-3x+x^2}$

$u = -3x+x^2$

$u' = -3+2x$

$\frac{dy}{dx} = (-3+2x)e^{-3x+x^2}$

18. $y = x^2 \cdot e^{-x^3}$ product rule

$2x \cdot e^{-x^3} + (-3x)e^{-x^3} \cdot x^2$

$2xe^{-x^3} - 3x^3e^{-x^3}$

19. $y = e^{-3/t^2}$

$u = \frac{-3}{t^2} = -3t^{-2}$

$u' = 6t^{-3}$

$\frac{dy}{dt} = 6t^{-3} \cdot e^{-3/t^2}$

$= \frac{6e^{-3/t^2}}{t^3}$

20. $f(x) = 4^x$

$\frac{d}{dx}[a^u] = (\ln a) \cdot a^u \cdot u'$

$a = 4$

$u = x$

$u' = 1$

$(\ln 4)(x)(1) = x \cdot \ln 4$

21. $y = \log_3 x$

$\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a) \cdot u}$

$a = 3$

$u = x$

$u' = 1$

$$\frac{1}{(\ln 3)(x)}$$

22. $y = x(6^{-2x})$ product rule

$$1 \cdot (6^{-2x}) + (\ln 6)(6^{-2x})(-2) \cdot x$$

$$6^{-2x} - 2x(\ln 6)(6^{-2x})$$

23. $\log_5 \frac{x^2-1}{x} = \log_5 (x^2-1) - \log_5 x$

$u = x^2-1$
$u' = 2x$
$a = 5$

$$\frac{2x}{(\ln 5)(x^2-1)} - \frac{1}{(\ln 5)(x)}$$

$u = x$
$u' = 1$
$a = 5$

$$\frac{x \cdot}{x \cdot}\frac{2x}{(\ln 5)(x^2-1)} - \frac{1}{(\ln 5)(x)}\frac{(x^2-1)}{(x^2-1)}$$

$$\frac{2x^2}{(\ln 5)(x)(x^2-1)} - \frac{(x^2-1)}{(\ln 5)(x)(x^2-1)}$$

$$\frac{x^2+1}{(\ln 5)(x)(x^2-1)}$$

24. $f(x) = \arcsin(x-1)$

$u = x-1$

$u' = 1$

$$\frac{u'}{\sqrt{1-u^2}} = \frac{1}{\sqrt{1-(x-1)^2}}$$

25. $y = 4\arctan\left(\frac{x}{8}\right)$

$u = \frac{1}{8}x$

$u' = \frac{1}{8}$

$$\frac{u'}{1+u^2} \qquad 4\left[\frac{\frac{1}{8}}{1+\left(\frac{x}{8}\right)^2}\right]$$

$$\frac{2}{1+\frac{x^2}{64}}$$

$26 \quad y = \underbrace{x^2}\cdot \underbrace{\arccos(\sqrt{x})}$ product rule

$u = x^{1/2}$

$u' = \frac{1}{2}x^{-1/2}$

$$2x \cdot \arccos\sqrt{x} + x^2 \cdot \left(-\frac{1}{2\sqrt{x}\sqrt{1-x}}\right) \qquad -\frac{u'}{\sqrt{1-u^2}}$$

$$2x\arccos\sqrt{x} - \frac{x^2}{2\sqrt{x}\sqrt{1-x}}$$

$$\frac{-\frac{1}{2\sqrt{x}}}{\sqrt{1-(\sqrt{x})^2}} = -\frac{1}{2\sqrt{x}}\cdot\frac{1}{\sqrt{1-x}}$$

$$= -\frac{1}{2\sqrt{x}\cdot\sqrt{1-x}}$$

27. $y = \frac{\operatorname{arccot}(2x)}{x}$ quotient rule

$\frac{-u'}{1+u^2}$ $u = 2x$
$u' = 2$

$$\frac{\frac{2}{1+(2x)^2} \cdot x - 1 \cdot \operatorname{arccot}(2x)}{x^2}$$

$$\frac{\frac{2x}{1+4x^2} - \operatorname{arccot}(2x)}{x^2} = \left(\frac{2x}{1+4x^2} - \operatorname{arccot}(2x)\right) \cdot \frac{1}{x^2}$$

$$\frac{2\cancel{x}}{\cancel{x^2}(1+4x^2)} - \frac{\operatorname{arccot}(2x)}{x^2}$$

$$\frac{2}{x(1+4x^2)} - \frac{\operatorname{arccot}(2x)}{x^2}$$