1. $3\ln x + 2\ln y - 4\ln z$

$\ln x^3 + \ln y^2 - \ln z^4$

positive logs on top

neg. logs on bottom

$$\ln\left(\frac{x^3y^2}{z^4}\right)$$

Properties of logs:

$\log a\cdot b = \log a + \log b$

$\log \frac{a}{b} = \log a - \log b$

$\log a^r = r\cdot \log a$

2) $2\left[\ln x - \ln(x+1) - \ln(x-1)\right]$

$$2\left[\ln\left(\frac{x}{(x+1)(x-1)}\right)\right]$$

$$\ln\left(\frac{x}{(x+1)(x-1)}\right)^2$$

3) $\frac{3}{2}\left[\ln(x^2+1)-\ln(x+1)-\ln(x-1)\right]$

$$\frac{3}{2}\left[\ln\left(\frac{x^2+1}{(x+1)(x-1)}\right)\right]$$

$$\ln\left(\frac{x^2+1}{(x+1)(x-1)}\right)^{3/2} \quad \text{or} \quad \ln\sqrt{\left(\frac{x^2+1}{(x+1)(x-1)}\right)^3}$$

4) $\ln \frac{xy}{z} = \ln x + \ln y - \ln z$

5) $\ln \left(\frac{x^2+1}{x^3}\right)^3 = 3\left[\ln(x^2+1) - 3\ln x\right]$

or $3\ln(x^2+1) - 9\ln x$

6) $\ln \sqrt[3]{a^2+1} = \ln(a^2+1)^{1/3} = \frac{1}{3}\ln(a^2+1)$

You can't separate a^2 & 1 because they are added, not multiplied.

7) $\ln(2x^2+1)$

Rule

$$\frac{d}{du}[\ln u] = \frac{1}{u} \cdot u'$$

$u = 2x^2+1$

$u' = 4x$

$$\frac{1}{2x^2+1} \cdot 4x = \frac{4x}{2x^2+1}$$

8) $\ln\sqrt{x^2-4}$

$u = x^2-4$

$u' = 2x$

$= \frac{1}{2}\ln(x^2-4)$

$= \frac{1}{2}\cdot\frac{1}{x^2-4}\cdot 2x = \frac{\cancel{2}x}{\cancel{2}(x^2-4)} = \frac{x}{x^2-4}$

9) $\ln\left(\dfrac{2x}{x+3}\right)$

$u = \dfrac{2x}{x+3}$

$u' = $ (quotient rule)

$$\frac{2(x+3) - 1(2x)}{(x+3)^2} = \frac{\cancel{2x}+6-\cancel{2x}}{(x+3)^2}$$

$$= \frac{6}{(x+3)^2}$$

$$\frac{1}{\frac{2x}{x+3}} \cdot \frac{6}{(x+3)^2}$$

$$1 \div \frac{2x}{x+3} = 1 \cdot \frac{x+3}{2x}$$

$$\frac{\cancel{x+3}}{\cancel{2}x} \cdot \frac{\cancel{6}\,3}{\cancel{(x+3)^2}\,(x+3)} = \frac{3}{x(x+3)}$$

9) $\ln\left(\frac{2x}{x+3}\right)$

ALTERNATE SOLUTION:

$$\ln 2x - \ln(x+3)$$

$$\frac{1}{\cancel{2}x}\cdot 2 - \frac{1}{x+3}\cdot 1$$

$$\frac{(x+3)}{(x+3)}\frac{1}{x} - \frac{1}{(x+3)}\cdot\frac{x}{x}$$ common denominator

$$\frac{\cancel{x}+3-\cancel{x}}{x(x+3)} = \frac{3}{x(x+3)}$$

10) $\dfrac{\ln t}{t}$ ← quotient rule

$$\frac{\frac{1}{t} \cdot t - 1 \cdot \ln t}{t^2} = \frac{\frac{t}{t} - \ln t}{t^2} = \frac{1 - \ln t}{t^2}$$

11) $\int \frac{5}{x}\,dx = 5\int \frac{1}{x}\,dx$

$= 5\left[\ln|x| + C\right]$

$= 5\ln|x| + C$

Rule:

$\int \frac{1}{u}\,du = \ln|u| + C$

12) $\int \frac{1}{x+2}\,dx$ $u = x+2$

$du = 1$

$\int \frac{1}{u}\,du = \ln|u| + C$

$= \ln|x+2| + C$

13) $$\int \frac{x^2-4}{x}\,dx$$

top power is bigger than bottom power so LONG DIVISION.

$$\begin{array}{r|l} & x - \frac{4}{x} \\ \hline x & x^2 + 0x - 4 \\ & -x^2 \\ \hline & 0 + 0x - 4 \leftarrow \text{remainder} \end{array}$$

$$\int \left(x - \frac{4}{x}\right) dx$$

$$\int x - \int \frac{4}{x}$$

$$\frac{1}{2}x^2 - 4\ln|x| + C$$

14) $\int \frac{x^2}{3-x^3}\,dx$ $\qquad u = 3-x^3$

$-\frac{1}{3}\,du = -\cancel{3}x^2$

$-\frac{1}{3}\int \frac{1}{u}\cdot du$

$-\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|3-x^3| + C$

15) $\displaystyle\int \frac{\csc^2 t}{\cot t}\,dt$

$u = \cot t$

$-1 \quad du = -\csc^2 t$

$-1\displaystyle\int \frac{1}{u}\,du = -1 \cdot \ln|u| + C$

$= -\ln|\cot t| + C$

16) $\int \frac{\cos x}{1+\sin x}$ $\qquad u = 1+\sin x$

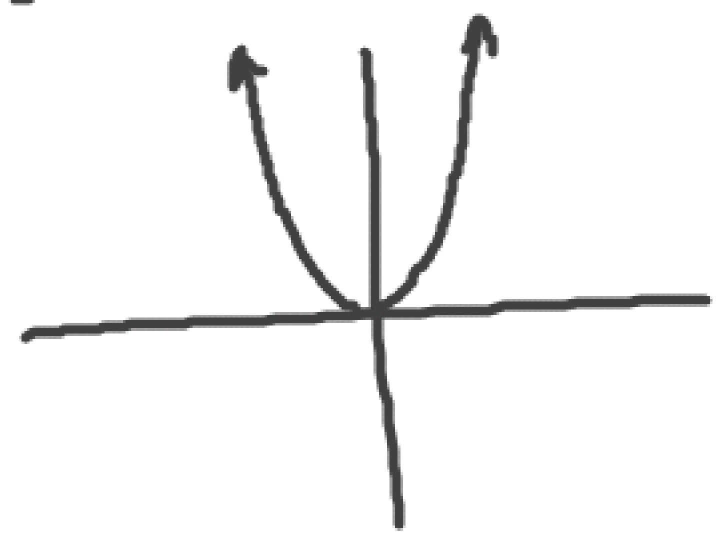
$\qquad\qquad\qquad\quad du = \cos x$

$$\int \frac{1}{u} \cdot du = \ln|u| + C$$

$$= \ln|1+\sin x| + C$$

17. $g(x) = x^2$

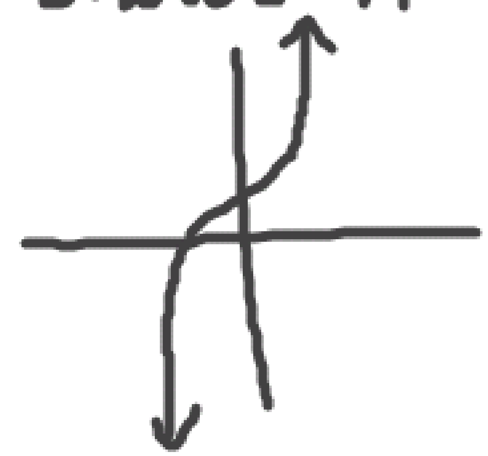
graphing x^2 shows it is not one-to-one so there is no inverse.



NOT ON TEST

18) $f(x) = x^3 + 1$

graph shows it is one-to-one



$x = y^3 + 1$

$-1 \qquad -1$

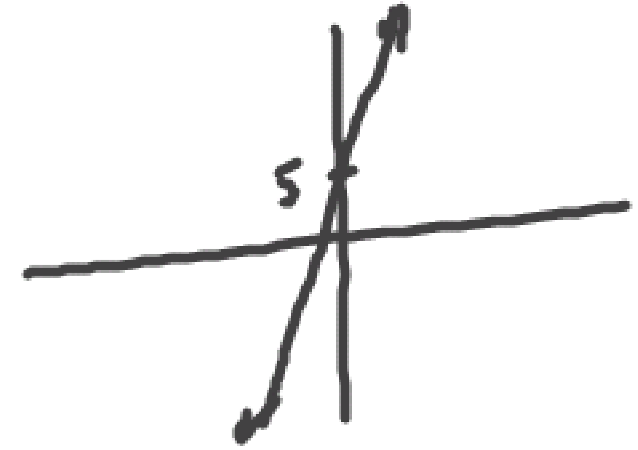
$\sqrt[3]{x-1} = \sqrt[3]{y^3}$

$\sqrt[3]{x-1} = f^{-1}(x)$

NOT ON TEST!

19) $h(x) = 2x + 5$

graph shows it is one-to-one



$$x = 2y + 5$$
$$-5 \qquad -5$$

$$\frac{x-5}{2} = \frac{2y}{2}$$

$$\frac{x-5}{2} = h^{-1}(x)$$

NOT ON TEST

20) $f(x) = e^{2x}$

$f'(x) = e^{2x} \cdot 2$

$= 2e^{2x}$

Rule:

$\frac{d}{du}[e^u] = e^u \cdot u'$

21) $y = e^{-3x+x^2}$

$u = -3x + x^2$

$u' = -3 + 2x$

$y' = e^{-3x+x^2}(-3+2x)$

$= (-3+2x)e^{-3x+x^2}$

22) $x^2 \cdot e^{-x^3}$ product rule

$= 2x \cdot e^{-x^3} + (e^{-x^3} \cdot -3x^2)\, x^2$

$= 2xe^{-x^3} + -3x^4 e^{-x^3}$

$= 2xe^{-x^3} - 3x^4 e^{-x^3}$

23) $y = e^{-3/t^2}$

$\frac{d}{dt}\left(\frac{-3}{t^2}\right) = -3t^{-2} = 6t^{-3}$

$= e^{-3/t^2} \cdot 6t^{-3}$

$= 6t^{-3}\left(e^{-3/t^2}\right)$

24.) $\int e^{-x^4}(-4x^3)\,dx$

Rule:

$\int e^u\,du = e^u + C$

$u = -x^4$

$du = -4x^3$

$\int e^u\,du = e^u + C$

$= e^{-x^4} + C$

25) $\int x^2 e^{x^3/2}\,dx$

$u = \frac{x^3}{2} = \frac{1}{2}x^3$

$\frac{2}{3}\,du = \cancel{\frac{3}{2}}x^2$

$\frac{2}{3}\int e^u\,du$

$\frac{2}{3}\left[e^u + C\right] = \frac{2}{3}e^{\frac{x^3}{2}} + C$

26)

$$\int \frac{e^{\frac{1}{x^2}}}{x^3}\,dx$$

$u = \frac{1}{x^2}$ or x^{-2}

$-\frac{1}{2}\,du = -2x^{-3}$ or $\cancel{\frac{-2}{x^3}}$

$$-\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C$$

$$= -\frac{1}{2}e^{\frac{1}{x^2}} + C$$

27) $f(x) = 4^x$

$= (\ln 4)(4^x) \cdot 1$

$= (\ln 4) 4^x$

Rule:

$\frac{d}{du}\left[a^u\right] = (\ln a) a^u \cdot u'$

28) $y = \log_3 x$

$$= \frac{1}{(\ln 3)\, x} \cdot 1$$

$$= \frac{1}{(\ln 3) x}$$

Rule:

$$\frac{d}{du}\left[\log_a u\right] = \frac{1}{(\ln a) \cdot u} \cdot u'$$

29) $x(6^{-2x})$ ← product rule

Rule:

$$\frac{d}{du}\left[a^u\right] = (\ln a)\, a^u \cdot u'$$

$$1 \cdot (6^{-2x}) + \left[(\ln 6)(6^{-2x})(-2)\right](x)$$

first factor

$$6^{-2x} - 2x(\ln 6)(6^{-2x})$$

30) $y = \log_5\left(\frac{x^2-1}{x}\right)$

Rule:

$$\frac{d}{du}[\log_a u] = \frac{1}{(\ln a)\cdot u}\cdot u'$$

separate into separate logs:

$$\log_5(x^2-1) - \log_5 x$$

$$\frac{1}{(\ln 5)(x^2-1)}\cdot 2x - \frac{1}{(\ln 5)x}\cdot 1$$

$$\frac{2x}{(\ln 5)(x^2-1)} - \frac{1}{(\ln 5)x}$$

31. $\int 3^x \, dx = \frac{1}{\ln 3} \cdot 3^x + c \qquad \int a^u \, du = \frac{1}{\ln a} a^u + c$

$a = 3$

$u = x$

$du = 1$

$$\frac{3^x}{\ln 3}$$

32. $\int x\left(5^{-x^2}\right)dx$

$a=5$

$u=-x^2$

$du=-2x$

$$-\frac{1}{2}\int a^u\,du = -\frac{1}{2}\cdot\frac{1}{\ln 5}5^{-x^2}+C$$

$$= -\frac{5^{-x^2}}{2x^2\ln 5}+C$$

33. $\int (3-x)\, 7^{(3-x)^2} dx$

$a = 7$

$u = (3-x)^2$

$du = 2(3-x)^1(-1)$

$= -2(3-x)$

$-\frac{1}{2}\int a^u \, du$

$-\frac{1}{2} \cdot \frac{1}{\ln 7} 7^{(3-x)^2} + C$

$= -\frac{7^{(3-x)^2}}{2\ln 7(3-x^2)} + C$