1. $\int (5-x)\,dx$

$5x - \frac{x^{1+1}}{2} = 5x - \frac{1}{2}x^2 + C$

2. $\int (4x^3+6x^2-1)\,dx$

$$\frac{4x^{3+1}}{4}+\frac{6x^{2+1}}{3}-1x+C$$

$$x^4+2x^3-1x+C$$

3. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx$

$\int \left(x^{\frac{1}{2}} + \frac{1}{2} x^{-\frac{1}{2}}\right) dx$

$\frac{x^{\frac{1}{2}+1}}{\frac{3}{2}} + \frac{1}{2} \frac{x^{-\frac{1}{2}+1}}{\frac{1}{2}} = \frac{2}{3} x^{3/2} + \cancel{\frac{1}{2} \cdot \frac{2}{1}} x^{1/2} + C$

$= \frac{2}{3} x^{3/2} + x^{1/2} + C$

4 $\int \left(\sqrt[4]{x^3} + 1\right) dx$

$$\int \left(x^{3/4} + 1\right) dx$$

$$\frac{x^{3/4+1}}{\frac{7}{4}} + 1x + C$$

$$= \frac{4}{7}x^{7/4} + x + C$$

5. $\int \frac{1}{x^4}\,dx$

$\int x^{-4}\,dx$

$\frac{x^{-4+1}}{-3} + C = -\frac{1}{3}x^{-3} + C$

or

$-\frac{1}{3x^3} + C$

6. $\displaystyle\int\left(\frac{x^2+2x-3}{x^4}\right)dx$ $\qquad \frac{x^2}{x^4}+\frac{2x}{x^4}-\frac{3}{x^4}$

$x^{-2}+2x^{-3}-3x^{-4}$

$$\int\left(x^{-2}+2x^{-3}-3x^{-4}\right)dx$$

$$\frac{x^{-2+1}}{-1}+\frac{2x^{-3+1}}{-2}-\frac{3x^{-4+1}}{-3}+C$$

$$-1x^{-1}-1x^{-2}+1x^{-3}+C$$

or $\displaystyle -\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}+C$

7. $\frac{1}{3(1)} + \frac{1}{3(2)} + \frac{1}{3(3)} + \frac{1}{3(4)} + \ldots + \frac{1}{3(9)}$

part that is changing from term to term so that is our "i".

ends with 9

$$\sum_{i=1}^{9} \frac{1}{3i} \quad \text{or} \quad \frac{1}{3}\sum_{i=1}^{9} \frac{1}{i}$$

starts w/ 1

8. $\left[1-\left(\frac{1}{4}\right)^2\right]+\left[1-\left(\frac{2}{4}\right)^2\right]+\ldots+\left[1-\left(\frac{6}{4}\right)^2\right]$

number over 4 is changing $\longrightarrow i$

$$\sum_{i=1}^{6}\left[1-\left(\frac{i}{4}\right)^2\right]$$

9. $3\left[\left(\frac{2}{n}\right)^3-\frac{2}{n}\right]+\ldots+3\left[\left(\frac{2n}{n}\right)^3-\frac{2n}{n}\right]$

Changes in two places, so the rule will have an i for both of them.

$$3\sum_{i=1}^{n}\left[\left(\frac{2i}{n}\right)^3-\frac{2i}{n}\right]$$

10. $4\sqrt{1-\frac{0}{n}} + \ldots + 4\sqrt{1-\frac{n-1}{n}}$

$$4\sum_{i=1}^{n}\sqrt{1-\frac{i-1}{n}}$$

11. $\sum_{i=1}^{15} 2i = 2 \cdot \sum_{i=1}^{15} i$

$= 2 \cdot \frac{15(16)}{2} = 240$

12. $\sum_{i=1}^{10} i^2 - 1 = \sum_{i=1}^{10} i^2 - \sum_{i=1}^{10} 1$

$= \frac{10(11)(21)}{6} - 1(10)$

$= 385 - 10 = 375$

13. $$\sum_{i=1}^{7} 4 = 4(7) = 28$$

14. $$\sum_{i=1}^{10} i(i^2+1) = \sum_{i=1}^{10} i^3+i = \sum_{i=1}^{10} i^3 + \sum_{i=1}^{10} i$$

$$= \frac{10^2 \cdot 11^2}{4} + \frac{10(11)}{2}$$

$$= 3025 + 55$$

$$= 3080$$

15. $\int_0^1 2x\,dx$

$\cancel{2}\frac{x^{1+1}}{\cancel{2}} = \left[x^2\right]_0^1 = 1^2 - 0^2 = 1 - 0 = 1$

16. $\int_{-1}^{0} (x-2)\,dx$

$\frac{x^{1+1}}{2} - 2x = \left[\frac{1}{2}x^2 - 2x\right]_{-1}^{0} = \left(\frac{1}{2}(0)^2 - 2(0)\right) - \left(\frac{1}{2}(-1)^2 - 2(-1)\right)$

$= 0 - \frac{5}{2} = -\frac{5}{2}$

17. $\int_{-1}^{1} (\sqrt[3]{t} - 2)\, dt$

$\int_{-1}^{1} (t^{1/3} - 2)\, dt$

$\dfrac{t^{1/3+1}}{4/3} - 2t = \left[\dfrac{3}{4} t^{4/3} - 2t\right]_{-1}^{1}$

$= \left(\frac{3}{4}(1)^{4/3} - 2(1)\right) - \left(\frac{3}{4}(-1)^{4/3} - 2(-1)\right)$

$= \left(-\frac{5}{4}\right) - \left(\frac{11}{4}\right) = -\frac{16}{4} = -4$

18. $\int_0^1 \left(t^{1/3} - t^{2/3}\right) dt$ ← should say dt not dx on practice test

$$\frac{t^{1/3+1}}{\frac{4}{3}} - \frac{t^{2/3+1}}{\frac{5}{3}} = \left[\frac{3}{4}t^{4/3} - \frac{3}{5}t^{5/3}\right]_0^1$$

$$= \left(\frac{3}{4}(1)^{4/3} - \frac{3}{5}(1)^{5/3}\right) - \left(\frac{3}{4}(0)^{4/3} - \frac{3}{5}(0)^{5/3}\right)$$

$$= \left(\frac{5\cdot 3}{5\cdot 4} - \frac{3\cdot 4}{5\cdot 4}\right) - (0-0)$$

$$= \frac{15}{20} - \frac{12}{20} = \frac{3}{20}$$

19.

$$\int_1^{27} \frac{x+1}{\sqrt{x}}\,dx$$

$$\int_1^{27} \left(\frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right) dx$$

$$\int_1^{27} \left(x^{1/2} + x^{-1/2}\right) dx$$

$$\left[\frac{2}{3}x^{3/2} + 2x^{1/2}\right]_1^{27}$$

$$\left(\frac{2}{3}(27)^{3/2} + 2(27)^{1/2}\right) - \left(\frac{2}{3}(1)^{3/2} + 2(1)^{1/2}\right)$$

$$(93.53 + 10.39) - \left(\frac{2}{3} + 2\right)$$

$$(103.92) - \left(\frac{8}{3}\right) = \boxed{101.253}$$

20. $\int_2^4 (\pi^2)\,dt$

π^2 is a constant:

$\int c\,dx = cx + C$

$\left[\pi^2 t\right]_2^4$

$4\pi^2 - 2\pi^2 = 2\pi^2$

21. $\int (x^2-9)^3 (2x)\,dx$ $\quad u = x^2-9$

$du = 2x$

$\int u^3\,du$

$\frac{u^{3+1}}{4} = \frac{1}{4}u^4 + C = \frac{1}{4}(x^2-9)^4 + C$

22. $\int t^3 \cdot \sqrt{t^4+5}\; dt \qquad u = t^4+5$

$$\frac{1}{4}\, du = \cancel{4}t^3$$

$$\frac{1}{4}\int u^{1/2}\, du$$

$$\frac{u^{1/2+1}}{\frac{3}{2}} = \frac{1}{4}\left(\frac{2}{3}u^{3/2}\right) = \frac{1}{6}u^{3/2} + C$$

$$= \frac{1}{6}\left(t^4+5\right)^{3/2} + C$$

23.

$$\int t\left(\sqrt[3]{t-4}\right)dt$$

$u = t-4$

$du = 1$

not accounted for in du!

$u = t-4$

$+4 \quad +4$

$u+4 = t$

Substitute for the t

$$\int (u+4)\cdot u^{1/3}\,du$$

$$\int u^{4/3+1} + 4u^{1/3+1}\,du$$

$$= \frac{4}{7}u^{7/3} + \cancel{4}\cdot\frac{3}{\cancel{4}}u^{4/3} + C$$

$$= \frac{4}{7}(t-4)^{7/3} + 3(t-4)^{4/3} + C$$

24. $\int_0^1 x\sqrt{1-x^2}\,dx$

$u = 1-x^2$

$-\frac{1}{2}\,du = -\cancel{2}x$

$-\frac{1}{2}\int_1^0 u^{1/2}\,du = -\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_1^0$

Don't forget to change the limits

$= -\frac{1}{2}\left(\frac{2}{3}(0)^{3/2} - \frac{2}{3}(1)^{3/2}\right)$

$= -\frac{1}{2}\left(0-\frac{2}{3}\right) = \frac{1}{3}$

25. $$\int_0^2 \frac{x}{\sqrt{1+2x^2}}\,dx = \int_0^2 x(1+2x^2)^{-1/2}\,dx$$

$$u = 1+2x^2$$

$$\frac{1}{4}\,du = \cancel{4}x$$

$$\frac{1}{4}\int_1^9 u^{-1/2}\,du$$

$$\frac{1}{4}\left[2u^{1/2}\right]_1^9 = \frac{1}{4}\left(2(9)^{1/2} - 2(1)^{1/2}\right)$$

$$= \frac{1}{4}(6-2) = \frac{1}{4}(4) = 1$$