1. $\lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

$$\lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 - 2x^2}{\Delta x} = \frac{2(x^2 + x\Delta x + x\Delta x + (\Delta x)^2) - 2x^2}{\Delta x}$$

$$\frac{2(x^2 + 2x(\Delta x) + (\Delta x)^2) - 2x^2}{\Delta x}$$

$$\frac{\cancel{2x^2} + 4x(\Delta x) + 2(\Delta x)^2 - \cancel{2x^2}}{\Delta x}$$

$$\frac{4x(\Delta x) + 2(\Delta x)^2}{\Delta x} = \frac{\cancel{\Delta x}(4x + 2(\Delta x))}{\cancel{\Delta x}}$$

$$\lim_{\Delta x \to 0} 4x + 2(\Delta x) = 4x$$

2. $$\lim_{\Delta x \to 0} \frac{(\sqrt{x+\Delta x} - 3) - (\sqrt{x} - 3)}{\Delta x} = \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}$$

multiply by conjugate of top to get rid of roots.

$$= \frac{(\sqrt{x+\Delta x} - \sqrt{x})(\sqrt{x+\Delta x} + \sqrt{x})}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})}$$

$$= \frac{\sqrt{x+\Delta x}^2 - \sqrt{x}^2}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})} = \frac{\cancel{x} + \cancel{\Delta x} - \cancel{x}}{\cancel{\Delta x}(\sqrt{x+\Delta x} + \sqrt{x})} = \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$\lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

3. $f(x) = -7$

$f'(x) = 0$ derivative of any constant $= 0$

4. $f(x) = 3x^4$

$f'(x) = 12x^3$ Power Rule

5. $g(x) = x^2 - 7x + 13$

$g'(x) = 2x - 7$

6. $y = \dfrac{1}{x^3}$

Change to an exponent:

$y = x^{-3}$

$\dfrac{dy}{dx} = -3x^{-4}$ Power Rule

7. $f(x) = 3x^4 - 2x^3 - 5x$

$f'(x) = 12x^3 - 6x^2 - 5$

8. $s(t) = 2t^3 - \cot t$

$s'(t) = 6t^2 - (-\csc^2 t)$

$= 6t^2 + \csc^2 t$

9. $f(x) = 2x^4 + 5x - 3x^{-4}$

$f'(x) = 8x^3 + 5 + 12x^{-5}$

or

$8x^3 + 5 + \frac{12}{x^5}$

10. $y = \sqrt[3]{x}$

Change to a power:

$y = x^{1/3}$

$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$ or $\frac{1}{3\sqrt[3]{x^2}}$

11. 2 ways to do it.

Product Rule:

$$g(x) = \underbrace{4x^2}_{1st}\underbrace{(x^2-2x)}_{second}$$

Product Rule:

$$8x(x^2-2x) + (2x-2)(4x^2)$$

$$8x^3 - 16x^2 + 8x^3 - 8x^2$$

$$g'(x) = \boxed{16x^3 - 24x^2}$$

Distribute then Power Rule:

$$g(x) = 4x^2(x^2-2x)$$

$$= 4x^4 - 8x^3$$

$$\boxed{g'(x) = 16x^3 - 24x^2}$$

12. $f(x) = \underbrace{x^4}_{1^{st}} \cdot \underbrace{\sin x}_{2^{nd}}$

$$4x^3 \cdot \sin x + \cos x\,(x^4)$$

$$4x^3 \sin x + x^4 \cos x$$

13. $g(t) = \dfrac{t^2-3}{4t+1}$ Quotient Rule

$$\frac{2t(4t+1) - 4(t^2-3)}{(4t+1)^2}$$

$$\frac{8t^2+2t-4t^2+3}{(4t+1)^2}$$

$$g'(t) = \frac{4t^2+2t+3}{(4t+1)^2}$$

14. $f(x)=\dfrac{\tan x}{x^2}$ Quotient Rule

$$\frac{\sec^2 x\,(x^2) - 2x\cdot\tan x}{(x^2)^2}$$

$$\frac{x^2\sec^2 x - 2x\tan x}{x^4}$$

$$\frac{\cancel{x}\,(x\sec^2 x - 2\tan x)}{\cancel{x^4}\ x^3}$$

$$f'(x)=\frac{x\sec^2 x - 2\tan x}{x^3}$$

15. $f(x) = 3x^3 + 2x$

$f'(x) = 9x^2 + 2$ Power Rule

16. $f(x) = \sqrt[3]{x}$

$= x^{1/3}$

$f'(x) = \frac{1}{3}x^{-2/3}$

13. $y = 2(1+3x^2)^3$

Chain Rule

$u = 1+3x^2$

$u' = 6x$

$y = 2u^3$

$y' = 6u^2 \cdot u'$

$= 6(1+3x^2)^2(6x)$

$= 36x(1+3x^2)^2$

18. $f(x) = \sqrt{2x+1}$

$u = 2x+1$

$u' = 2$

$f(u) = \sqrt{u} = u^{1/2}$

$f'(u) = \frac{1}{2} u^{-\frac{1}{2}} \cdot u'$

$f'(x) = \frac{1}{\cancel{2}} (2x+1)^{-\frac{1}{2}} \cdot \cancel{(2)}$

$= (2x+1)^{-\frac{1}{2}}$

-or-

$= \frac{1}{\sqrt{2x+1}}$

19. $g(x) = \left(\dfrac{3x}{4x+1}\right)^2$

$u = \dfrac{3x}{4x+1}$

$g(u) = u^2$

$g'(u) = 2u \cdot u'$

$u' =$ quotient rule

$u' = \dfrac{3(4x+1) - 4(3x)}{(4x+1)^2}$

$u' = \dfrac{\cancel{12x} + 3 - \cancel{12x}}{(4x+1)^2}$

$u' = \dfrac{3}{(4x+1)^2}$

$g'(x) = 2\left(\dfrac{3x}{4x+1}\right) \cdot \dfrac{3}{(4x+1)^2}$

$$\boxed{= \dfrac{18x}{(4x+1)^3}}$$

20.

$$g(x) = 2\sec(3x^2) \qquad u = 3x^2$$

$$u' = 6x$$

$$g(u) = 2\sec u \cdot u'$$

$$g'(u) = 2\sec u \tan u \cdot u'$$

$$= 2\sec(3x^2)\tan(3x^2) \cdot 6x$$

$$= 12x\sec(3x^2)\tan(3x^2)$$

21. $y = 2\sin^2(3x+2)$

note: $\sin^2 x = (\sin x)^2$

$y = 2\left(\sin(3x+2)\right)^2$

$u = \sin(3x+2)$

$u' =$ chain rule again

$\sin(\underbrace{3x+2}_{t})$

$t = 3x+2$

$t' = 3$

$\sin t$

$\frac{du}{dt} = \cos t \cdot t'$

$\cos(3x+2)(3)$

$u' = 3\cos(3x+2)$

$= 2u^2$

$\frac{dy}{du} = 4u \cdot u'$

$= 4\underbrace{\left(\sin(3x+2)\right)}_{u} \cdot \underbrace{3\cos(3x+2)}_{u'}$

$\boxed{= 12\sin(3x+2)\cdot\cos(3x+2)}$

22. $f(x) = \tan\left(\sqrt{x^2+1}\right)$

$u = \sqrt{x^2+1}$

$u' =$ chain rule again

$\tan u$

$t = x^2+1$

$f'(u) = \sec^2 u \cdot u'$

$t' = 2x$

$t^{1/2}$

$f'(x) = \sec^2\sqrt{x^2+1} \cdot \left(\dfrac{x}{\sqrt{x^2+1}}\right)$

$\frac{1}{2}t^{-\frac{1}{2}} \cdot t'$

$$\boxed{= \frac{x\sec^2\sqrt{x^2+1}}{\sqrt{x^2+1}}}$$

$\cancel{\frac{1}{2}}(x^2+1)^{-\frac{1}{2}} \cdot \cancel{2}x$

$x(x^2+1)^{-\frac{1}{2}} = \dfrac{x}{\sqrt{x^2+1}}$

* Don't cancel the roots, the one on top is stuck inside the $\sec^2$ function.

$u' = \dfrac{x}{\sqrt{x^2+1}}$