1. $\sqrt{25}$

Factor 25

$25 = 5 \cdot 5$

$5 \cdot 5$ 1 pair of 5's

$\sqrt{25} = 5$

2. $\sqrt{x^4}$

you can get 2 pairs out of 4 x's

$(x \cdot x) \cdot (x \cdot x)$

$\sqrt{x^4} = x^2$

3. $\sqrt[3]{y^6}$

you can get 2 groups of 3 out of 6 y's.

$(y \cdot y \cdot y)(y \cdot y \cdot y)$

$\sqrt[3]{y^6} = y^2$

4. $\sqrt{81a^6b^{12}c^2}$

$= 9a^3b^6c$

$\sqrt{81} = 9$

$\sqrt{a^6} = a^3$

$\sqrt{b^{12}} = b^6$

$\sqrt{c^2} = c$

5. $\sqrt{\frac{4x^{10}}{25}} = \frac{\sqrt{4x^{10}}}{\sqrt{25}} = \frac{2x^5}{5}$

6. $\sqrt[4]{16x^4y^{12}}$

$2xy^3$

$16 = 2 \cdot 2 \cdot 2 \cdot 2$

$2 \quad 2 \quad 2 \quad 2$

7. $\sqrt{32x^2y^{11}}$

$32 = (2\cdot 2)(2\cdot 2)2$

$2 \quad 16$

$2 \quad 8$

$2 \quad 4$

$2 \quad 2$

$2^2\sqrt{2}\cdot x\cdot y^5\sqrt{y}$

$4xy^5\sqrt{2y}$

8. $\sqrt{75a^5b^{12}c^7}$

$75 = (5\cdot 5)3$

$3 \quad 25$

$5\cdot 5$

$5\sqrt{3}\cdot a^2\sqrt{a}\cdot b^6\cdot c^3\sqrt{c}$

$5a^2b^6c^3\sqrt{3ac}$

9. $(5+\sqrt{3})(4-\sqrt{2})$

$20-5\sqrt{2}+4\sqrt{3}-\sqrt{6}$

10. $(\sqrt{5}+\sqrt{2})^2 = (\sqrt{5}+\sqrt{2})(\sqrt{5}+\sqrt{2})$

$= \sqrt{25}+\sqrt{10}+\sqrt{10}+\sqrt{4}$

$= 5+2\sqrt{10}+2$

$= 7+2\sqrt{10}$

11. Can only add radicals that have the same index & radicand.

$\sqrt{5} + \sqrt{20} + \sqrt{45}$ ① simplify each radical

$\sqrt{5} + 2\sqrt{5} + 3\sqrt{5}$ ② add numbers in front

one ↗

$6\sqrt{5}$

12. $\sqrt{27} - \sqrt{8} - \sqrt{12} + \sqrt{32}$

$3\sqrt{3} - 2\sqrt{2} - 2\sqrt{3} + 4\sqrt{2}$

$1\sqrt{3} + 2\sqrt{2}$

13. $\dfrac{4}{3+\sqrt{2}}$ $(3-\sqrt{2})$ $(3-\sqrt{2})$

multiply the top & bottom by the conjugate of the bottom

$$\frac{12-4\sqrt{2}}{9-3\sqrt{2}+3\sqrt{2}-\sqrt{4}} = \frac{12-4\sqrt{2}}{7}$$

$$9-2=7^2$$

14. $\dfrac{2+\sqrt{3}}{4-\sqrt{3}} \cdot \dfrac{(4+\sqrt{3})}{(4+\sqrt{3})} = \dfrac{8+2\sqrt{3}+4\sqrt{3}+\sqrt{3}}{16-4\sqrt{3}+4\sqrt{3}-\sqrt{3}}$

$16-3 \qquad 3$

$= \dfrac{11+6\sqrt{3}}{13}$

15. $5^{\frac{1}{3}}$ ← power, ← index

$\sqrt[3]{5^1} = \sqrt[3]{5}$

16. $x^{\frac{3}{5}} = \sqrt[5]{x^3}$

17. $\sqrt[3]{x^2} = x^{\frac{2}{3}}$

18. $\sqrt[3]{5x^2y} = (5x^2y)^{\frac{1}{3}} = 5^{\frac{1}{3}} x^{\frac{2}{3}} y^{\frac{1}{3}}$

19. $x^{\frac{1}{3}} \cdot x^{\frac{2}{5}}$ add the powers

$x^{\frac{5}{15}} \cdot x^{\frac{6}{15}} = x^{\frac{5}{15}+\frac{6}{15}} = x^{\frac{11}{15}}$

20. $\frac{y^{\frac{2}{3}}}{y^{\frac{1}{4}}} = y^{\frac{2}{3}-\frac{1}{4}} = y^{\frac{8}{12}-\frac{3}{12}} = y^{\frac{5}{12}}$

21. $\left(a^{\frac{3}{4}}\right)^{\frac{2}{3}} = a^{\frac{3}{4}\cdot\frac{2}{3}} = a^{\frac{6}{12}} = a^{\frac{1}{2}}$

22. $\dfrac{4y^{\frac{1}{2}}}{y^{\frac{1}{6}}\cdot y^{\frac{1}{3}}} = \dfrac{4y^{\frac{1}{2}}}{y^{\frac{1}{6}+\frac{1}{3}}} = \dfrac{4y^{\frac{1}{2}}}{y^{\frac{1}{6}+\frac{2}{6}}} = \dfrac{4y^{\frac{1}{2}}}{y^{\frac{3}{6}}} = \dfrac{4y^{\frac{1}{2}}}{y^{\frac{1}{2}}}$

$= 4y^{\frac{1}{2}-\frac{1}{2}} = 4y^{0} = 4$

23. $\sqrt{x+5} \geq 4$

Find the restriction:

$x+5 \geq 0$

$-5 \quad -5$

$x \geq -5$

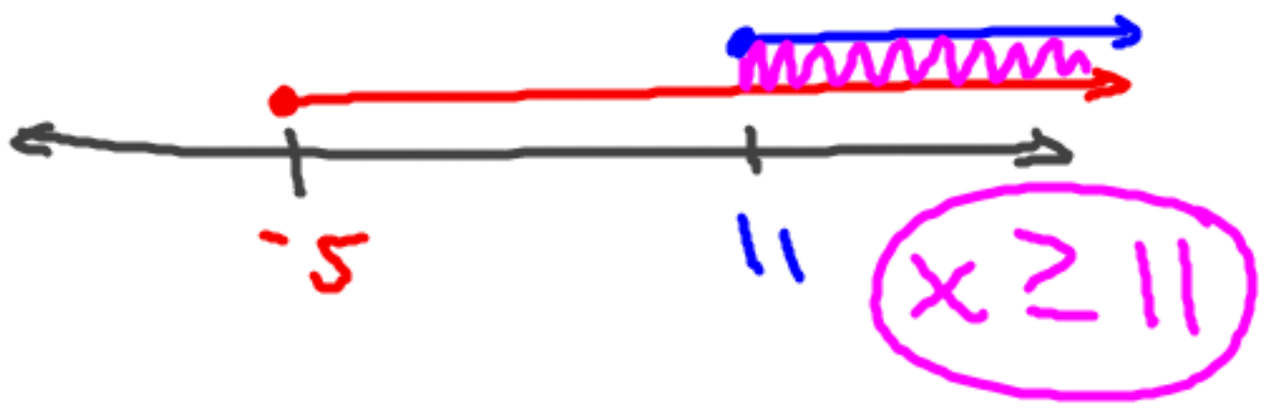
Solve:

$\sqrt{x+5}^2 \geq 4^2$

$x+5 \geq 16$

$-5 \quad -5$

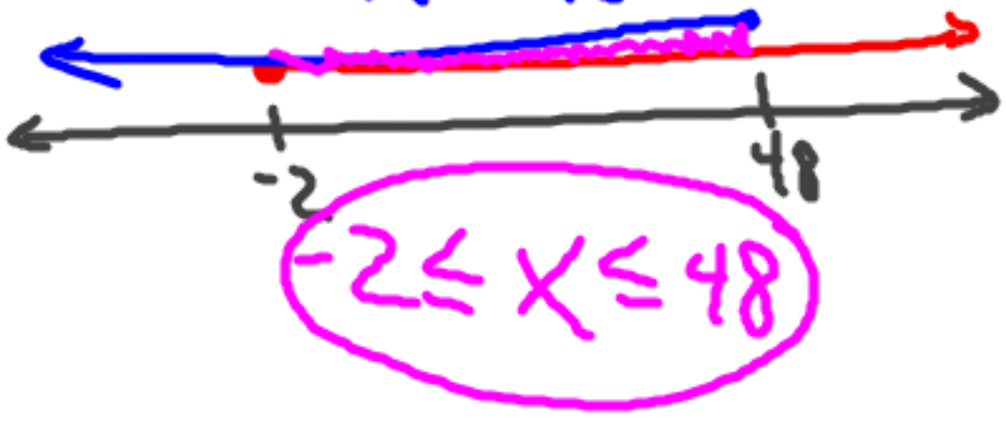
$x \geq 11$



24. $\sqrt{2x+4} - 3 \leq 7$

$2x+4 \geq 0$
$-4 \quad -4$
$\frac{2x}{2} \geq \frac{-4}{2}$
$x \geq -2$

$\sqrt{2x+4} - 3 \leq 7$
$+3 \quad +3$
$(\sqrt{2x+4})^2 \leq 10^2$
$2x+4 \leq 100$
$-4 \quad -4$
$\frac{2x}{2} \leq \frac{96}{2}$
$x \leq 48$



$-2 \leq x \leq 48$

25. $\sqrt{x-3}^2 = \sqrt{2x-1}^2$

$x - 3 = 2x - 1$

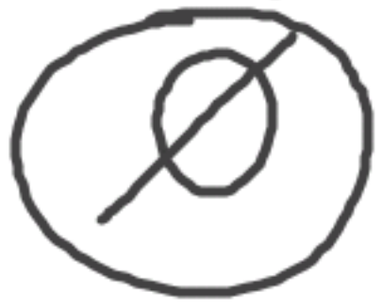
$+1 \quad +1$

$-x \quad -x$

$-2 = x$

Check: $\sqrt{-2-3} = \sqrt{2(-2)-1}$

$\sqrt{-5} = \sqrt{-5}$ can not have negatives inside a square root so:



26. $(3x+4)^{\frac{1}{2}}+3=7$

$-3 \quad -3$

$\left((3x+4)^{\frac{1}{2}}\right)^2 = 4^2$

$3x+4=16$

$-4 \quad -4$

$\frac{3x}{3} = \frac{12}{3}$

$x=4$

Check:

$(3(4)+4)^{\frac{1}{2}}+3=7$

$(12+4)^{\frac{1}{2}}+3=7$

$(16)^{\frac{1}{2}}+3=7$

$4+3=7$

$7=7$ ✓